# Eight Queens Write-up (Sean)

## Overview

Eight Queens is a well-known problem where one tries to place eight queens strategically on a standard 8x8 chessboard in positions where none are able to attack the other. According to the standard rules of chess, a queen can attack any piece that is the same column, row, or diagonal. Therefore, the key is to place the eight queens so that none are in the same row, column, or diagonal. A brute force algorithm testing every possible grid alignment takes quite a while because the grid is 8 by 8, but the algorithm can be simplified by first preventing queens from taking the same row or column as the other.

## History

According to an article by Paul Campbell, the origin of the “eight queens” problem lies in 1848 with Max Bezzel, a German chess player who proposed the concept in *Schachzeitung*. A variety of solutions were proposed but the problem received little publicity. Franz Nauck then rewrote the problem in 1850 into a more widely-circulated newspaper called *Illustrirte Zeitung* of Leipzig, and Gauss, a renowned mathematician, read the problem and sent it to an astronomer friend of his, H.C. Schumacher. Without going into too much detail, the two went back and forth debating the number of possible solutions. However, by the end, both agreed that there were over 70 solutions. The maximum number suggested was 92, but neither could prove for certain that this was valid. Other mathematicians worked on the problem in parallel such as Lionnet and Bellavitis. Nauck was also the one who proposed expanding the problem beyond just 8 queens and suggested the “n queens problem”, where one tries to place *n* queens on a chessboard of n by n squares.

## Algorithm

There are a massive number of possible placements for an 8 by 8 grid (according to Wikipedia, 4,426,165,368) but only 92 solutions to the eight queens problem. However, this massive number can be narrowed somewhat by placing constraints such as preventing placing the queens in the same column or row ahead of time. This reduces the possible to 88 solutions. There are further limits that can be put in place including checking diagonals.

## N-Queens and Further Extension

As discussed in the history section, the eight queens problem can be expanded more generally to placing n queens in an n by n board. If one does this, interesting patterns emerge. First, there are no solutions for a 2x2 or 3x3 grid, and there are more unique solutions for a 5x5 grid than there are for a 6x6 grid. However, once a 7x7 grid is created, it at this point that the sheer number of unique solutions becomes huge. At 25x25, there are over 270 trillion unique solutions (meaning solutions that are not rotations or reflections of each other). This becomes increasingly difficult for computers to process due to the sheer quantity of possible solutions.

The n-queens problem can be expanded further by introducing a variety of other elements. One such element is the inclusion of unmovable queens that occupy set positions which the computer must move around. Another possible extension is the so-called super-queen problem where queens can also attack like a knight. This addition means that there are no solutions for boards up to 10x10.

## Example Solution

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